Diffusers for wind turbines

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1 Introduction

The mass flow through a wind turbine rotor can be increased by an expanding duct also called a diffuser. However, to be effective, a good diffuser must have a certain geometry. There are wind turbines on the market with diffusers for which I think that they have hardly any positive effect on the increase of the mass flow. To explain this, first it is necessary to understand what happens with the air flow for a wind turbine with no diffuser. This is explained in chapter 4.2 of my report KD 35 (ref. 1) for a horizontal axis wind turbine (HAWT). But this theory also works for a vertical axis wind turbine (VAWT). Chapter 4.2 of KD 35 is now copied but the numbering of the formulas and pictures is changed.

Not all the kinetic energy can be extracted from the wind because this would result in the reduction of the wind speed to zero and this implies that no air is flowing through the rotor area. It was Betz who developed a theory in 1926 which gives the absolute maximum value for the power coefficient C_p.

The pressure just before the windmill rotor is somewhat higher than the pressure just behind the rotor if the rotor extracts energy from of the wind. This pressure difference forms a resistance to the air flowing through the rotor. Therefore the wind has a tendency to flow around the rotor instead of through it.

The air particles flowing through the rotor form a stream-tube. This stream-tube is bounded by air particles which just touch the blade tip. The distance in between the stream line of these particles and the rotor axis increases as a particle is followed from a position far upstream from the rotor up to a position far downstream. This means that the stream-tube is expanding. The shape of the stream-tube is illustrated in figure 1. Theoretically far upstream and far downstream the rotor means at an infinite distance from the rotor but in practice a distance of about three rotor diameters is enough to describe the principle.

The cross sectional area of the stream-tube far in front of the rotor is labelled A_1. The swept area of the rotor is labelled A_r. The cross sectional area of the stream-tube far behind the rotor is labelled A_2. At A_1 the wind speed is undisturbed. This undisturbed wind speed is labelled V. Betz showed that maximum power is extracted from the wind if the wind speed at A_r = 2/3 V and if the wind speed at A_2 is 1/3 V. If the air is considered incompressible, the product of the wind speed and the area must be constant so:

$$A_1 * V = A_r * 2/3\ V = A_2 * 1/3\ V. \qquad (1)$$

This results in $A_1 = 2/3\ A_r$ and $A_2 = 2\ A_r$.

Betz used momentum theory and considered the windmill rotor as a so-called actuator disk. This is simply a mechanism which extracts energy from the wind. It needs not necessarily to be a rotating windmill rotor with blades.

The maximum power P_{max}, which the rotor can extract from the wind if all losses are neglected, is the power at A_1 minus the power at A_2. The power in the wind P_w is given by formula 2.5 of KD 35 which is copied as formula 2.

$$P_w = ½\rho V^3 * A \qquad (W) \qquad (2)$$

Combining formula 2 and $A_1 = 2/3\ A_r$ and $A_2 = 2\ A_r$ we get:

$$P_{max} = ½\rho V^3 * 2/3\ A_r - ½\rho\ (1/3V)^3 * 2\ A_r \text{ or}$$

$$P_{max} = 16/27 * ½\rho V^3 * A_r \qquad (W) \qquad (3)$$

The power available in the undisturbed wind at A_r is given by formula 2.

The ratio in between the maximum power which can be extracted from the wind at A_r and the power in the undisturbed wind at A_r (no rotor in place), is called the Betz coefficient. Combining formula 3 and 2 we get:

Betz coefficient = P_{max} / P_w = 16/27 = 0.59 (-) (4)

The pressure difference over the rotor area results in a resulting thrust force F_t acting on the centre of the rotor. F_t is given by:

$F_t = C_t * ½ \rho V^2 * \pi R^2$ (N) (5)

Betz found that C_t = 8/9 if the wind speed at the rotor plane is reduced to 2/3 V. For a real windmill C_t generally is somewhat lower than the Betz value. For the rotor of the VIRYA-3.3 windmill, which is used in the examples (given in chapter 5.4 of KD 35), C_t is about 0.7 for $\lambda = \lambda_{opt}$. If the rotor is running unloaded, so for $\lambda = \lambda_{unl}$, C_t is about the same. C_t is much lower if the rotor is not running, that is $\lambda = 0$. For this condition C_t is determined by the combined drag on all the blades. C_t for $\lambda = 0$ is much lower for rotors with a high λ_{opt} than for rotors with a low λ_{opt} because rotors with a high λ_{opt} have a much lower total blade area (see KD 35 chapter 5). This is the end of the copied chapter 4.2 out of KD 35.

The real $C_{p\ max}$ of a wind turbine rotor is always smaller than the Betz coefficient because of four reasons: wake rotation, tip losses, airfoil drag and a real blade length k shorter than R. The first three reasons are explained in chapter 4.3 of KD 35. The fourth reason is explained by formula 6.3 of KD 35.

Figure 4.1 out of KD 35 gives only an impression of the expanding wake but isn't drawn to scale. Figure 1 of this report KD 600 is drawn to scale. It is assumed that the rotor at A_r has a diameter D. The distance in between A_1 and A_r and between A_r and A_2 is chosen 3 * D. As A_1 = 2/3 A_r it can be calculated that D_1 = 0.8165 D.
As A_2 = 2 A_r it can be calculated that D_2 = 1.4142 D.

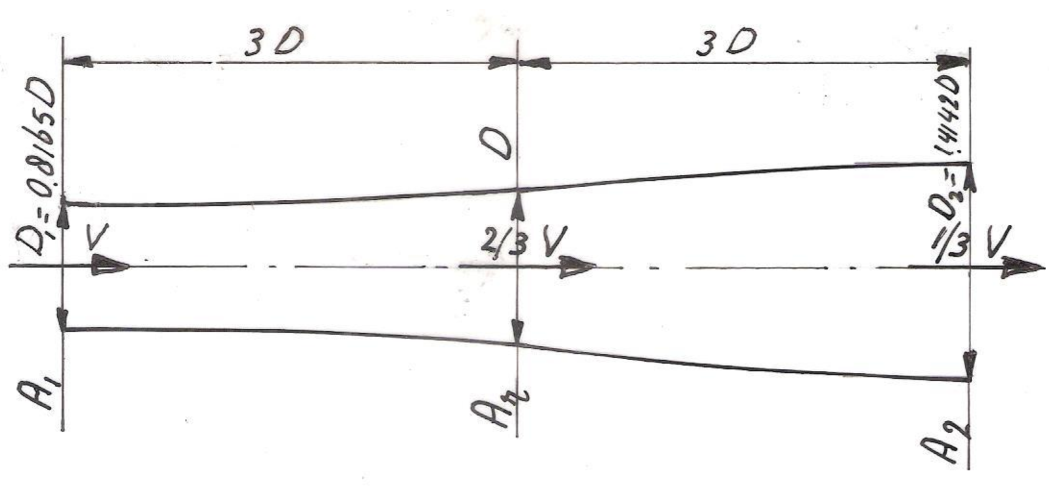

Fig. 1 Stream-tube of expanding air flowing through a windmill rotor at maximum power

So figure 1 shows on scale what happens with the air flow through a rotor if the maximum power is extracted from the wind. So this means that if a diffuser would be made which has the shape of the given stream-tube, or of only a part of it just around the rotor, it will have absolutely no effect as the stream-tube isn't changed!

For the calculation of the areas A_1 and A_2 as a function of A_r, the air is considered incompressible. This is allowed if the pressure difference over the rotor plane is very low. The formula for the pressure difference Δp can be derived from formula 5 which gives the rotor thrust F_t, by dividing F_t by the swept area A. This gives:

$$\Delta p = C_t * \tfrac{1}{2}\rho V^2 \qquad (N/m^2) \qquad (6)$$

Assume $C_t = 0.7$, $\rho = 1.2$ kg/m^3 and V = 10 m/s. Substitution of these values in formula 6 gives Δp = 42 N/m^2. The atmospheric pressure is about 1 bar = 100000 N/m^2. So the pressure difference over the rotor plane at V = 10 m/s is only a factor 0.00042 of the atmospheric pressure and therefore it is allowed to consider the air incompressible.

The pressure difference in between the front and the back side of a rotor blade is much higher than the calculated pressure difference over the whole rotor plane. The pressure difference over a blade is larger as the solidity of the rotor is smaller. However, for the shape of the expanding wake only the average pressure difference over the whole rotor plane counts.

2 Ducted wind turbines

To get an increase of the mass flow both areas A_1 and A_2 have to be increased. Next assume that one wants an increase of the mass flow by a factor 4. This means that both areas A_1 and A_2 have to increase by a factor 4 and so the diameters have to increase by a factor 2. This means that D_1 becomes 2 * 0.8165 = 1.633 D and that D_2 becomes 2 * 1.4142 = 2.8284 D.

A diffuser which meets these requirements is drawn in figure 2. It must be prevented that the air flowing outside the diffuser is stalling at the entrance of the diffuser and therefore the entrance is rounded. To prevent sudden changes in the wind speed, the diffuser must be rather long. It is assumed that the part of the diffuser before the rotor must have a length of at least D and that the part behind the rotor must have a length of at least 3 D. So an effective diffuser will be very large compared to the rotor diameter D.

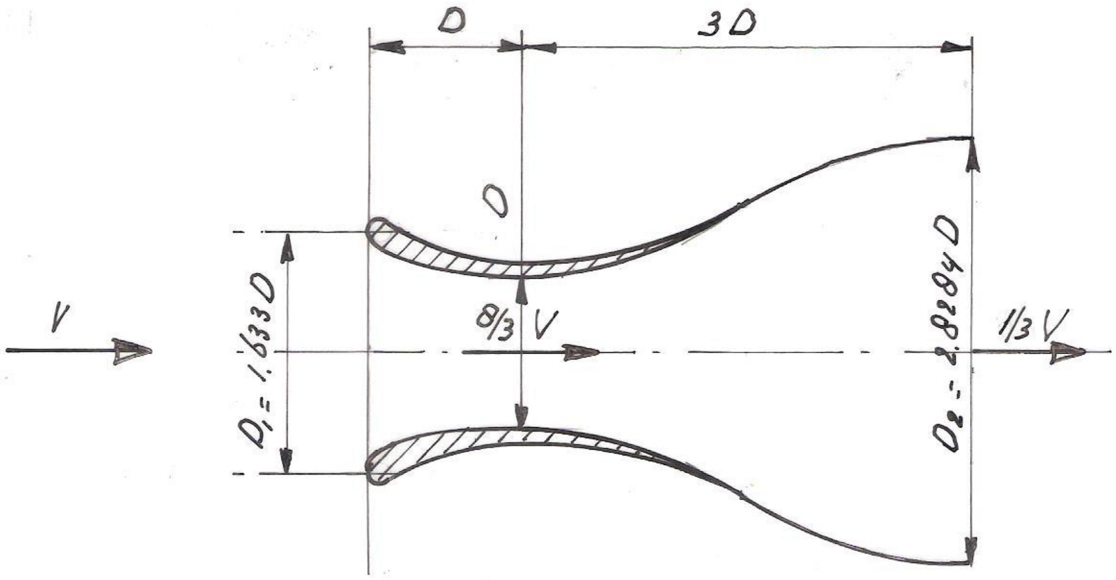

Fig. 2 Effective diffuser

If the diffuser is that effective that the mass flow through the rotor is increased by a factor 4, this means that the wind speed in the rotor plane is also increased by a factor 4 and so it becomes 4 * 2/3 V = 8/3 V. As the powers at D_1 and D_2 are a factor 4 higher than for a rotor with the same diameter D but without a diffuser, it means that the power which can be extracted with the given diffuser will be a factor 4 higher than with no diffuser.

For a non ducted wind turbine the power goes up with the cube of the undisturbed wind speed V. As the wind speed in the rotor plane is a factor 2/3 of V it also goes up with the cube of wind speed in the rotor plane. But for a ducted wind turbine this last relation is no longer valid as it appears that increase of the wind speed in the rotor plane by a factor 4 results in increased of the power by only a factor 4 and not by a factor $4^3 = 64$!

This is only possible if the pressure difference over the rotor plane is not increased by the diffuser. But this means that one can't use the normal rotor design theory to design a rotor for a ducted wind turbine. The rotor for a ducted wind turbine must be designed for much lower lift coefficients. So one can't simply take an existing rotor and built a diffuser around it!

The functioning of the diffuser is not only determined by the air flow in the diffuser but also by the air flow outside the diffuser. The air flow outside the diffuser is compressed in between point 1 and point 2. So the wind speed outside the diffuser is larger at point 2 than at point 1. This increase of the wind speed results in decrease of the pressure. So the pressure at point 2 is lower than at point 1. This under pressure at point 2 is sucking the inside flow through the diffuser.

So an effective diffuser is rather big if compared to the rotor diameter D. The investment in such a very large diffuser will be very high. The same increase in power will also be realised if the rotor diameter D would be increased by a factor 2. I think that the higher costs of a rotor with the double diameter will be less than the extra costs of a large diffuser and that this is the reason why effective large diffusers are not used.

The diffuser has to follow the wind direction but because of its shape, it works like a vane. But therefore it is not possible to use a safety system for protection of too high rotational speeds and thrust, which turns the rotor out of the wind.

Most ducted wind turbines have diffuser dimensions with only a little difference in diameter in between D_1 and D_2 and these diffusers therefore will give almost no increase of the mass flow. What they do is that they prevent tip losses which flow from the front site to the back side of a rotor blade. However, figure 4.3 of KD 35 shows that the reduction of the C_p because of the tip losses for a rotor with 3 blades and a design tip speed ratio of about 7 is only about 0.03. So the increase of the power coefficient realised by a small diffuser which only reduces the tip losses, is very small. The extra investment costs of a small diffuser which only realises this effect will be higher than the extra costs of a slightly larger rotor which gives the same increase of power. And even a small diffuser will work as a vane so one can't use turning the rotor out of the wind as a safety system. So the main advantage of these small diffusers is that something is available which looks nice.

In stead of a diffuser which is mainly located behind the rotor, sometimes people come with the idea of a concentrator which is positioned before the rotor. I think that the effect of a concentrator with a certain maximum diameter is much smaller that that of a diffuser with the same maximum diameter. Another option is to simply place a disc around the rotor. This disk will result in some extra pressure difference over the rotor plane but it will also cause a lot of turbulence behind the disk. The extra power which can be generated with a disk with the same diameter as the diameter D_2 of the diffuser of figure 2 will be much lower than that of a well shaped diffuser.